

# Quench dynamics and counting statistics in interacting nanojunctions: Quasi-particles trapping



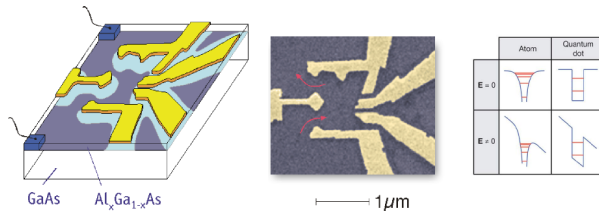
Rubén Seoane Souto

Normal case: R  mi Avriller, Rosa Carmina Monreal,   lvaro Mart  n  
Rodero and Alfredo Levy Yeyati

Superconducting:   lvaro Mart  n Rodero and Alfredo Levy Yeyati

September 20, 2016

Systems with confined electrons: quantum dots, nanowires, single molecules,...



Several applications: high efficient electronic devices, medicine,...

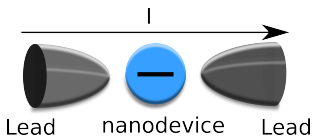
Problem of quantum transport through a quantum dot

- Quench dynamics
- Counting statistics

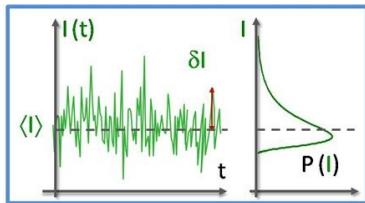
Role of interactions

# Introduction(I): Why time-dependent?

## Electronic transport through nanoscaled devices



## electron transport properties

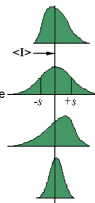


**First cumulant**  
mean current

**Second cumulant:**  
Standard deviation. Shot noise

**Third cumulant**  
Skewness

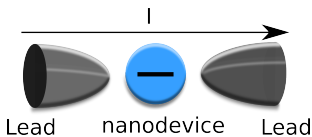
**Fourth cumulant**  
Kurtosis



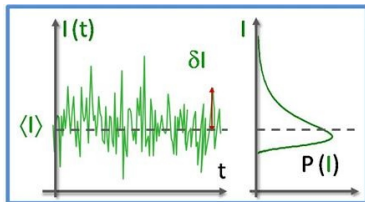
## Time-dependent evolution?

# Introduction(I): Why time-dependent?

## Electronic transport through nanoscaled devices



## electron transport properties

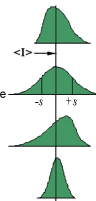


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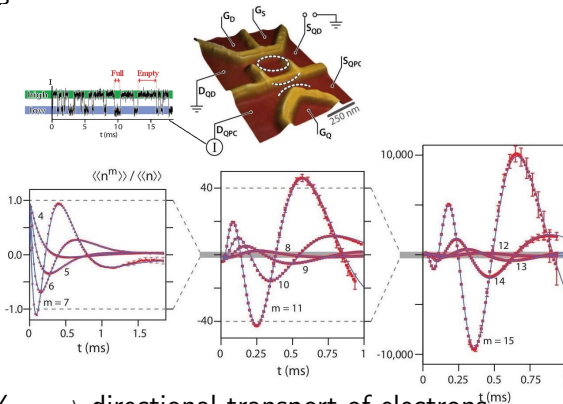
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Kurtosis



## Time-dependent evolution?



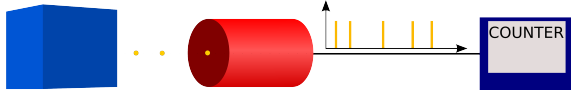
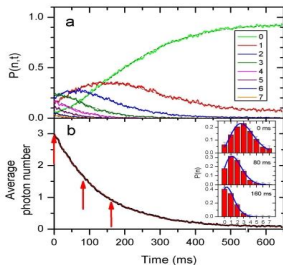
Transient regime: Flindt et al.<sup>†</sup>



- Large  $V_{bias} \rightarrow$  directional transport of electrons
- Sequential regime
- Non-interacting model

<sup>†</sup>PNAS, 10116 (2009)

Counting photons statistics  $\rightarrow$  since 70's.

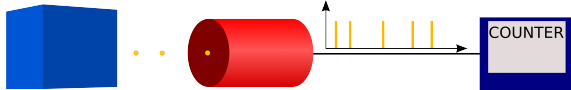
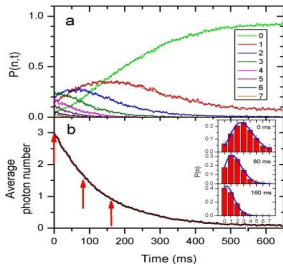


t-evolution probabilities of number of photons<sup>†</sup>

What about electrons?

<sup>†</sup>Phys. Rev. Lett. **101**, 240402 (2008)

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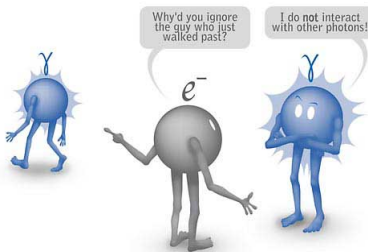
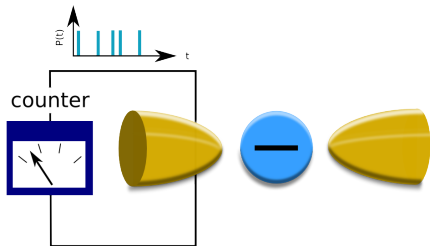
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## Electron Counting Statistics

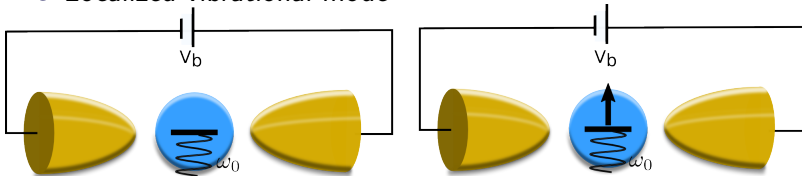
- Detector Coupled to the System
- No directional flux of electrons
- **Role of interactions**



- 1 Localized vibrational mode
  - Anderson-Holstein model
  - Generating function
  - Results
- 2 Superconducting junctions
  - Model Hamiltonian
  - Results
- 3 Conclusions

Anderson-Holstein comprises different kind of nanojunctions:  
single molecule transistors, nanowires, carbon nanotubes,...

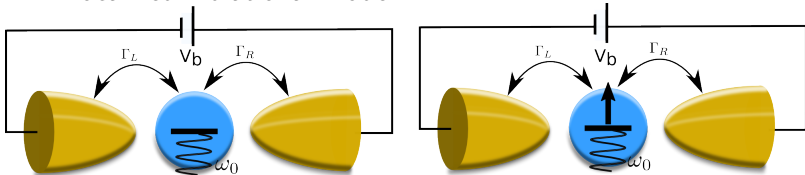
- Single spinless level
- Localized vibrational mode



$$H = H_{Leads} + \epsilon_0 d^\dagger d + \omega_0 a^\dagger a + \lambda d^\dagger d (a + a^\dagger)$$

Anderson-Holstein comprises different kind of nanojunctions:  
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- Localized vibrational mode



$$H = H_{Leads} + \epsilon_0 d^\dagger d + \omega_0 a^\dagger a + \lambda d^\dagger d (a + a^\dagger) + H_T \theta(t), \quad \Gamma_i = \pi \rho_{Fi} t_i^2 \quad (i = L, R)$$

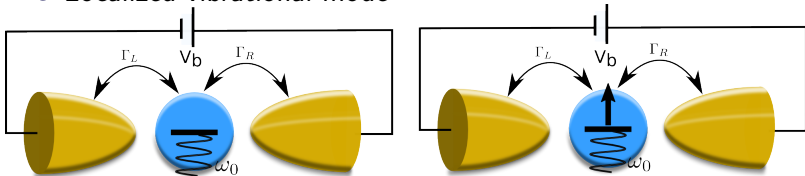
Generating function

$$Z(\chi, t) = \sum_n P_n(t) e^{i\chi n}$$

$$\langle c_n(t) \rangle = \left. \frac{\partial^n \text{Log}(Z(\chi, t))}{\partial \chi^n} \right|_{\chi=0}$$

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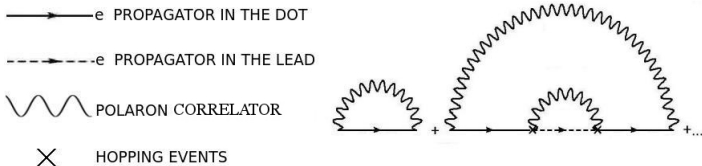
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## Dressed Tunneling Approximation: Diagrammatic resummation of dominant Feynman diagrams<sup>†</sup>



- Equilibrium population of phonons

<sup>†</sup>R. Seoane Souto et. al. Phys. Rev. B **89**, 085412 (2014)

Generating function:

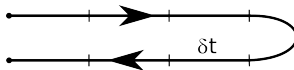
$$Z(\chi, t) = \left\langle T_c e^{i \int_c H_{T\chi} dt} \right\rangle$$

Functional determinant  $\rightarrow$  Discretized in time contour<sup>†</sup>

$$Z(\chi, t) = \frac{\det \left[ \hat{g}_0^{-1}(t) - \hat{\Sigma}(\chi, t) \right]}{\det \left[ \hat{g}_0^{-1}(t) - \hat{\Sigma}(\chi = 0, t) \right]}$$

$\hat{g}_0 \rightarrow$  Green function of the dot

$\hat{\Sigma} \rightarrow$  Self-energy (coupling to the electrodes)



<sup>†</sup>Rev. Mod. Phys. **81** (2009)

Generating function:

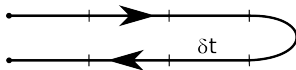
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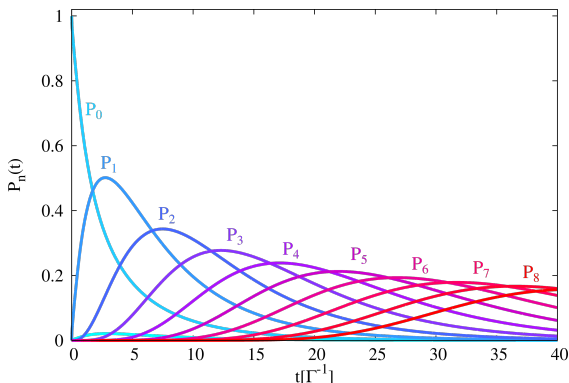
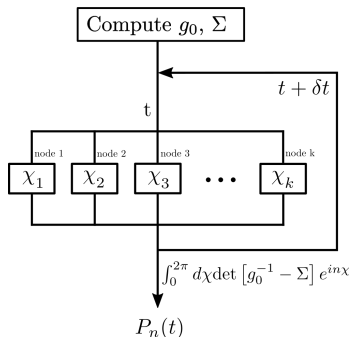


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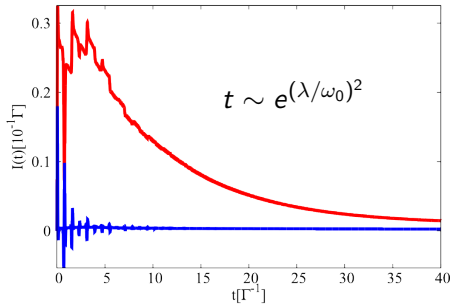
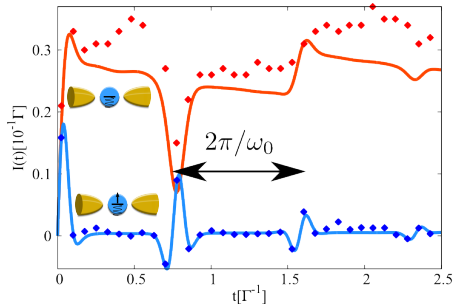
Probability of electrons transferred

$$P_n = \int_0^{2\pi} d\chi Z(\chi, t) e^{in\chi} = \frac{\int_0^{2\pi} d\chi \det \left[ \hat{g}_0^{-1}(t) - \hat{\Sigma}(\chi, t) \right] e^{in\chi}}{\det \left[ \hat{g}_0^{-1}(t) - \hat{\Sigma}(\chi = 0, t) \right]}$$

Strong scaling  $\approx 1$



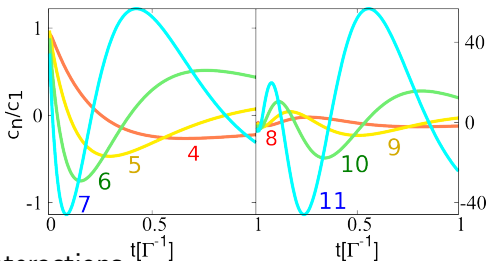
## Current evolution. Quantum Monte Carlo data<sup>†</sup>



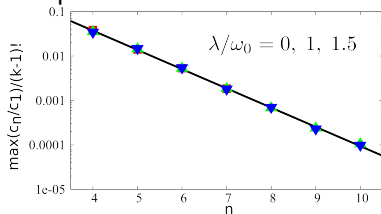
- Bistability
- Polaronic effects causes long transient behavior

<sup>†</sup>K. F. Albretch et al. Phys. Rev. B **87**, 085127 (2013)

Universal oscillatory behavior at short times. Experiments <sup>†</sup>



independent of interactions



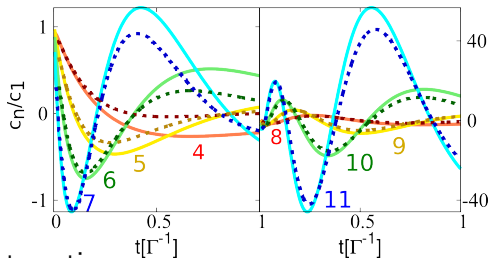
$$\langle c_n(t) \rangle = \left. \frac{\partial \text{Log}[Z(\chi, t)]}{\partial \chi^n} \right|_{\chi=0}$$

$$Z(\chi, t) = c_1(t) (e^{i\chi} - 1)$$

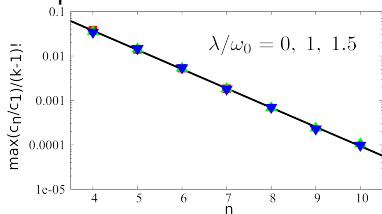
$$\max(c_n/c_1) = (n-1)! e^{-\pi}$$

<sup>†</sup>PNAS, 10116 (2009)

Universal oscillatory behavior at short times. Experiments <sup>†</sup>



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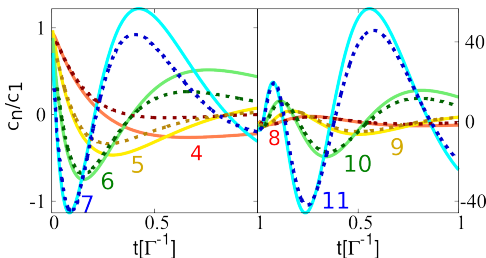
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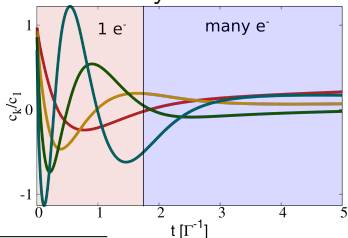
$$\max(c_n/c_1) = (k-1)! e^{-\pi}$$

<sup>†</sup>PNAS, 10116 (2009)

## Universal oscillatory behavior at short times. Experiments



Many electrons  $\rightarrow$  Broken universality  $^\dagger$

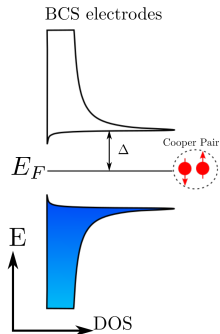
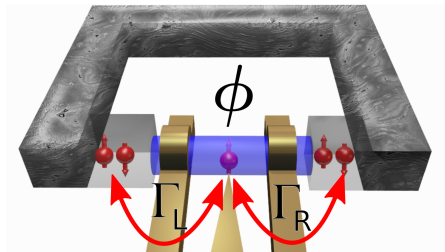


$^\dagger$ R. Seoane et al. PRB 92 (12) 125435, 2015 ;Accepted in Fortschritte der Physik



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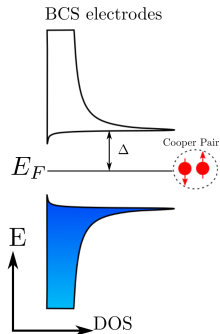
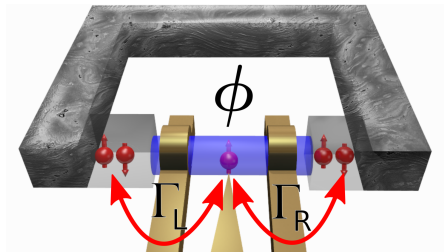
## Phase driven superconducting junction



$\phi \rightarrow$  Magnetic flux through the loop

$$H = H_{BCS \text{ Leads}} + \epsilon_0 \sum_{\sigma=\uparrow,\downarrow} d_{\sigma}^{\dagger} d_{\sigma} + H_T \theta(t)$$

## Phase driven superconducting junction

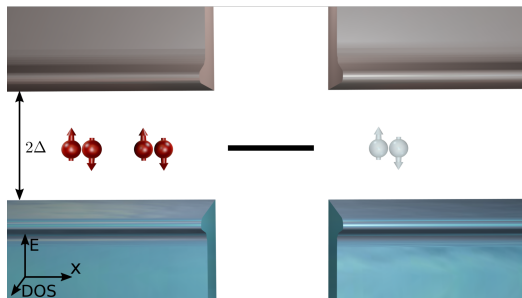


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electron-hole reflections  $\rightarrow$  Andreev reflections

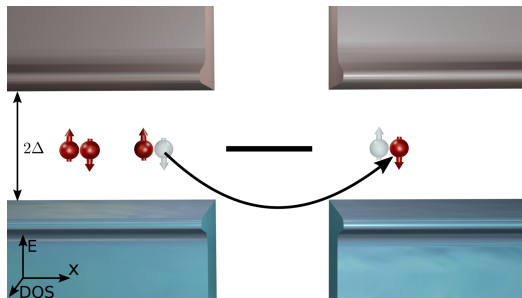
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Multiple Andreev reflections  $\rightarrow$  Andreev Bound states

electron-hole reflections  $\rightarrow$  Andreev reflections

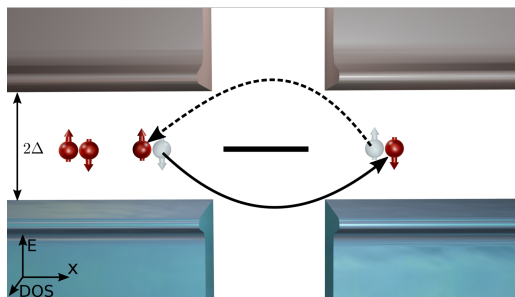
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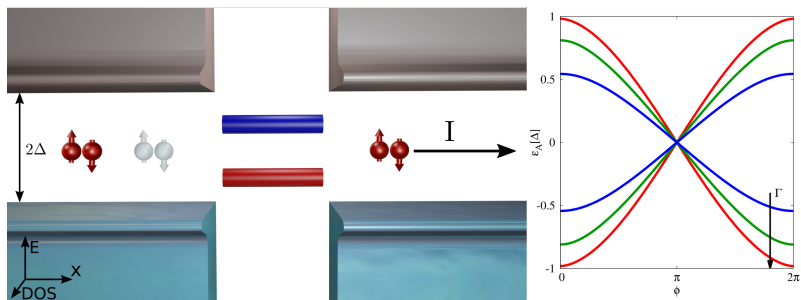
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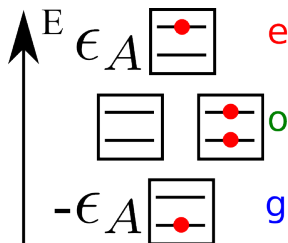
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Multiple Andreev reflections  $\rightarrow$  Andreev Bound states

Transport dominated by the presence of the Andreev Bound States<sup>†</sup>

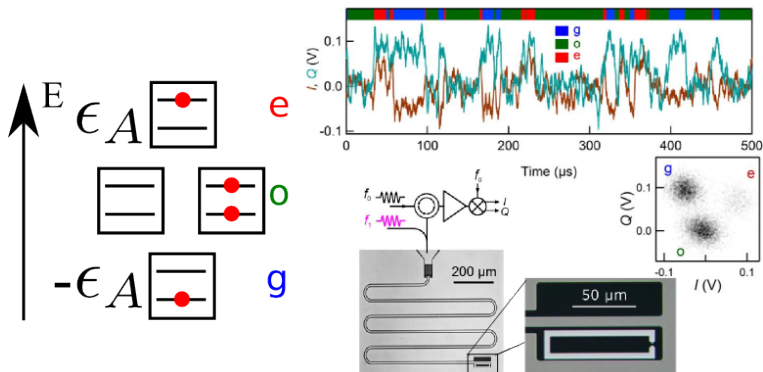


$I(t) \propto n_{\text{ground}}(t) - n_{\text{excited}}(t)$   
 $T=0$ : only the *ground* state is populated.

<sup>†</sup>Science **349**, 1199 (2015)



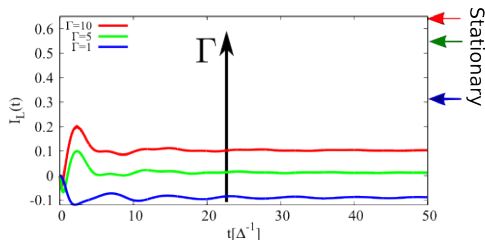
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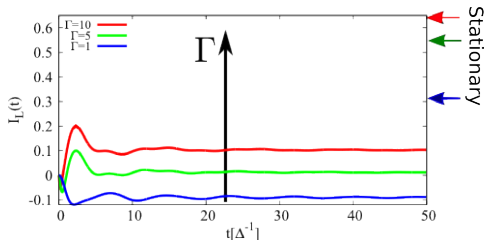
<sup>†</sup>Science **349**, 1199 (2015)

Current is not converging to the stationary value (even with adiabatic switching)

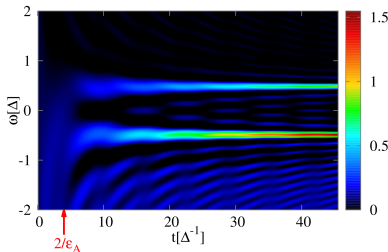


Non-interacting Andreev Bound states  $\rightarrow$  non-equilibrium population

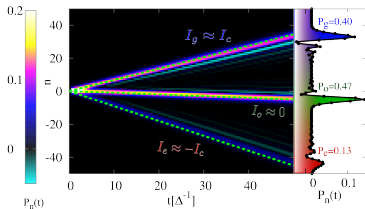
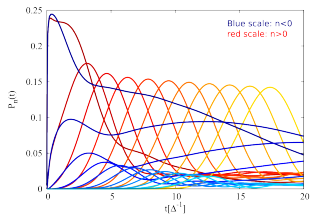
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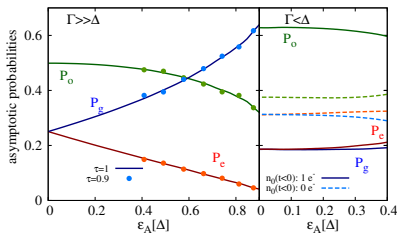
Non-interacting Andreev Bound states  $\rightarrow$  non-equilibrium population



Counting statistics  $\rightarrow$  contributions from the four Fock states

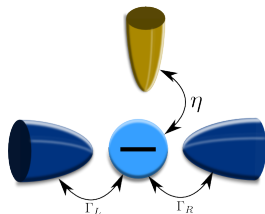


Quasi-particles trapped in the system. Universal behavior at  $\Gamma \gg \Delta^\dagger$



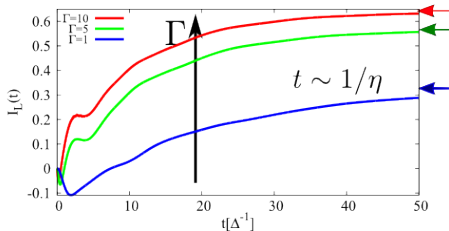
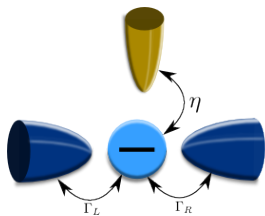
$^\dagger$ Submitted to Phys. Rev. Lett.

Relaxation mechanism: weak coupling to a third normal electrode



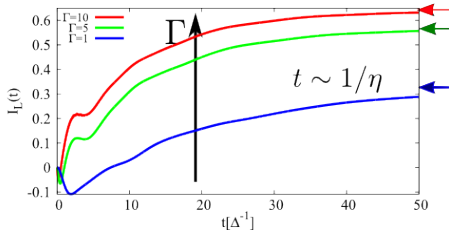
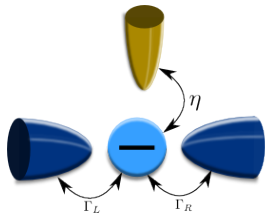
Population of the excited state relaxes

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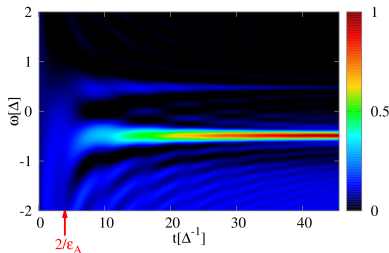


Population of the excited state relaxes

Relaxation mechanism: weak coupling to a third normal electrode



Population of the excited state relaxes



## Electron counting statistics in transient regime

- Holstein model
  - Long transient dynamics
  - Universal oscillations in higher current cumulants
- Superconducting electrodes
  - Non-equilibrium population of Andreev bound states
  - Extra relaxation mechanism to reach stationary

## Future

- Stationary regime
- Coherent control of the system  $\rightarrow$  Andreev qubit



## Computational resources



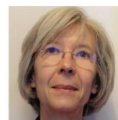
## Funding



## Our group in Madrid



Alfredo Levy



Rosa C. Monreal



Álvaro Martín

## Bordeaux



Rémi Avriller

# Thank you!